Aufgabe 1:

a) rg (A) = 2

$$\begin{array}{rcrcl} x_1 & & & = & 2\cdot x_3 - 3\cdot x_4 - x_5 \\ 2\cdot x_1 & - & 3\cdot x_2 & = & -2\cdot x_3 \qquad\qquad - 2\cdot x_5 \end{array}$$

$x_3 = 1 \quad ; \quad x_4 = x_5 = 0 \quad \Rightarrow \quad x_1 = 2$

$2\cdot x_1 - 3\cdot x_2 = -2 \quad \Rightarrow \quad x_2 = 2$

$$x^{(1)} = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$x_3 = 0 \quad ; \quad x_4 = 1 \quad ; \quad x_5 = 0 \quad \Rightarrow \quad x_1 = -3$

$2\cdot x_1 - 3\cdot x_2 = 0 \quad \Rightarrow \quad x_2 = -2$

$$x^{(2)} = \begin{pmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$x_3 = x_4 = 0 \quad ; \quad x_5 = 1 \quad \Rightarrow \quad x_1 = -1$

$2\cdot x_1 - 3\cdot x_2 = -2 \quad \Rightarrow \quad x_2 = 0$

$$x^{(3)} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$x = \alpha_1 \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 \cdot \begin{pmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_3 \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

b) rg(A) =2 ==> nur die triviale Lösung $x = \mathit{o}$

c) rg(A) = 2

$$\begin{array}{lcccccc} x_1 & = & -3x_3 & + & 2x_4 & - & x_5 \\ x_2 & = & -x_3 & + & x_4 & - & 5x_5 \end{array}$$

$x_3 = 1 \quad ; \quad x_4 = x_5 = 0 \quad \Rightarrow \quad x_1 = -3 \quad ; \quad x_2 = -1$

$$x^{(1)} = \begin{pmatrix} -3 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$x_3 = 0 \quad ; \quad x_4 = 1 \quad ; \quad x_5 = 0 \quad \Rightarrow \quad x_1 = 2 \quad ; \quad x_2 = 1$

$$x^{(2)} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$x_3 = x_4 = 0 \quad ; \quad x_5 = 1 \quad \Rightarrow \quad x_1 = -1 \quad ; \quad x_2 = -5$

$$x^{(1)} = \begin{pmatrix} -1 \\ -5 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$x = \alpha_1 \cdot \begin{pmatrix} -3 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_3 \cdot \begin{pmatrix} -1 \\ -5 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

d) rg(A) =2

$$\begin{array}{rcrcr} x_1 & - & 3x_2 & = & -2x_3 \\ 4x_1 & - & 2x_2 & = & -5x_3 \end{array}$$

$$x_3 = 1 \quad \Rightarrow \quad \begin{array}{rcrcr} x_1 & - & 3x_2 & = & -2 \\ 4x_1 & - & 2x_2 & = & -5 \end{array} \quad \Rightarrow \quad x_2 = 0{,}3 \quad \Rightarrow \quad x_1 = -1{,}1 \quad \Rightarrow \quad x = \alpha \cdot \begin{pmatrix} -1{,}1 \\ 0{,}3 \\ 1 \end{pmatrix}$$

<u>Aufgabe 2:</u>

$$\begin{vmatrix} 3 & 0 & -3 \\ -1 & 2 & 3 \\ a & 3 & 1 \end{vmatrix} = 3 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} + (-3) \cdot \begin{vmatrix} -1 & 2 \\ a & 3 \end{vmatrix} = -21 + 9 + 6a = -12 + 6a = 0 \quad \Rightarrow \quad a = 2$$

Für a = 2 existieren nichttriviale Lösungen!!