# Lösungen für 15.Übung Mathematik Sommersemester

**Aufgabe 1:**

a) $F(x) = \frac{5}{7}x^7 - \frac{1}{3}x^6 + x^3 - 7x + c$

b) partielle Integration: $\begin{matrix} u' = x^4 & u = \frac{1}{5}x^5 \\ v = \ln x & v' = \frac{1}{x} \end{matrix} \quad \Rightarrow \quad F(x) = \frac{1}{5}x^5 \cdot \ln|x| - \frac{x^5}{25} + c$

c) partielle Integration:

$\begin{matrix} u' = e^x & u = e^x \\ v = \sin x & v' = \cos x \end{matrix} \quad \Rightarrow \quad \int e^x \cdot \sin x\, dx = e^x \cdot \sin x - \int e^x \cdot \cos x\, dx$

*erneute partielle Integration für* $\int e^x \cdot \cos x\, dx$ *liefert*:

$\begin{matrix} u' = e^x & u = e^x \\ v = \cos x & v' = -\sin x \end{matrix} \quad \Rightarrow \quad \int e^x \cdot \cos x\, dx = e^x \cdot \cos x + \int e^x \cdot \sin x\, dx$

$\Rightarrow \quad 2\int e^x \cdot \sin x\, dx = e^x \cdot \sin x - e^x \cdot \cos x + c$

$\Rightarrow \quad \int e^x \cdot \sin x\, dx = 0{,}5 \cdot e^x \cdot (\sin x - \cos x) + c$

d) $\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$

$$\int \cot^2 x\, dx = \int \frac{dx}{\sin^2 x} - \int dx = -\cot x - x + c$$

e) $F(u) = u \cdot \cot^2 x + c$ ( da die Integrationsvariable nicht x sondern u ist!!!! )

**Aufgabe 2:**

a)

$\begin{matrix} p' = 1 & p = u \\ q = \cos(\ln u) & q' = \frac{-\sin(\ln u)}{u} \end{matrix} \Rightarrow \int \cos(\ln u)\, du = u \cdot \cos(\ln u) + \int \sin(\ln u)\, du$

erneute partielle Integration:

$\begin{matrix} p' = 1 & p = u \\ q = \sin(\ln u) & q' = \frac{\cos(\ln u)}{u} \end{matrix} \quad \Rightarrow \quad \int \sin(\ln u)\, du = u \cdot \sin(\ln u) - \int \cos(\ln u)\, du$

$\Rightarrow \quad 2\int \cos(\ln u)\, du = u \cdot \left(\cos(\ln u) + \sin(\ln u)\right) + c$

$\Rightarrow \quad \int \cos(\ln u)\, du = 0{,}5 \cdot u \cdot \left(\cos(\ln u) + \sin(\ln u)\right) + c$

b)

$u' = z^n \qquad u = \frac{z^{n+1}}{n+1}$

$v = \ln z \qquad v' = \frac{1}{z}$

$$\Rightarrow \quad \int z^n \cdot \ln z \, dz = \frac{z^{n+1}}{n+1} \cdot \ln z - \int \frac{z^n}{n+1} dz = \frac{z^{n+1}}{n+1} \cdot \ln z - \frac{z^{n+1}}{(n+1)^2} + c = \frac{z^{n+1}}{n+1} \cdot \left( \ln z - \frac{1}{n+1} \right) + c$$